8th All India RPS OLYMPIAD

Online & Offline Mega Event

QUESTION PAPER

CLASS - X

RPS MATHEMATICS LEAGUE

Q1. If the radius of a circle is decreased by 50%, the percentage decrease in its area is:

(A) 50% (B) 75% (C) 25% (D) 100%

Q2. The area of a regular hexagon inscribed in a circle of radius r is:

(A) $\frac{3\sqrt{3}\,r^2}{2}$ (B) $\frac{3r^2}{2}$ (C) $\sqrt{3}r^2$ (D) $3\sqrt{3}r^2$

Q3. If $in\theta + cos\theta = \sqrt{2}cos\theta$, then the value of $tan\theta$ is:

(A) $\sqrt{2} - 1$ (B) $\sqrt{2} + 1$ (C) $\sqrt{2}$ (D) 1

Q4. Two circles of radii 10 cm and 8 cm intersect each other and the common chord is 12 cm. The distance between their centres is:

(A) $\sqrt{7} + \sqrt{5}$ (B) $2\sqrt{7} + 2\sqrt{5}$ (C) $8 + 2\sqrt{7}$ (D) $2\sqrt{10} + 2\sqrt{7}$

Q5. If the vertices of a triangle are $(1, \sqrt{3})$, (0, 0) and (2, 0), the centroid of the triangle is:

(A) $(1, \frac{1}{\sqrt{3}})$ (B) $(1, \frac{\sqrt{3}}{2})$ (C) $(1, \frac{1}{2\sqrt{3}})$ (D) $(\frac{3}{2}, \frac{\sqrt{3}}{2})$

Q6. The interior angles of a polygon are in A.P. If the smallest angle is 120° and the common difference is 5°, the number of sides of the polygon is:

(A) 9 (B) 16 (C) 9 or 16 (D) 12

Q7. Coordinate of a point on Y-axis which is equidistance from the points (6,5) and (-4,3) are

(A) (3, 2) (B) (0,0) (C) (0,9) (D) (9,0)

Q8. If the roots of $x^2 - px + q = 0$ differ by 1, then:

(A) $p^2 = 4q + 1$ (B) $p^2 = 4q - 1$

(C) $q^2 = 4p + 1$ (D) $p^2 + q^2 = 1$

Q9. The number of ordered pairs of positive integers satisfying $\frac{1}{x} + \frac{1}{y} = \frac{1}{12}$ is:

(A) 11 (B) 12 (C) 15 (D) 18

Q10. If p and q are prime numbers such that $p^2 - q^2 = 960$, then the value of $p + q$ can be:

(A) 64 (B) 68 (C) 72 (D) 96

Q11. Let α and β are the zeroes of f(x)= $x^2 - x + 1$, then value of $\alpha^4 + \beta^4$

(A) -1 (B) 2 (C) -2 (D) +1

Q12. Let n be a positive integer. If $n^5 - 5n^3 + 4n$ is always divisible by an integer k for all $n \geq 3$, find the greatest value of k.

(A) 60 (B) 120 (C) 240 (D) 720

Q13. If $x + y + z = 1$, $x^2 + y^2 + z^2 = 2$, and $x^3 + y^3 + z^3 = 3$, then the value of $x^4 + y^4 + z^4$ is:

(A) $\frac{25}{6}$ (B) $\frac{13}{2}$ (C) $\frac{17}{3}$ (D) $\frac{19}{4}$

Q14. $(1 + tan1^\circ)(1 + tan2^\circ) \ldots\ldots (1 + tan44^\circ)$

(A) 2^{44} (B) 2^{23} (C) 2^{22} (D) 2^{21}

Q15. If one root of $x^2 - 3x + k = 0$ is twice of the other, then k is?

(A) 3 (B) −3 (C) 2 (D) −2

Q16. For what value(s) of k does $(k - 2)x^2 + 2(k - 2)x + 2 = 0$ have equal real roots?

(A) {2, 4} (B) Only 4 (C) 2, 3 (D) Only 2

Q17. The system of equations $kx + y = k^2$ and $x + ky = 1$ has infinitely many solutions if:

(A) k = 1 (B) k = −1

(C) k = 2 (D) No such value of k exists

Q18. Find the area of the region bounded by the lines $2x + 3y = 6, 2x - y = 2$ and the y-axis.

(A) 3 sq. units (B) 4 sq. units (C) 6 sq. units (D) 8 sq. units

Q19. If the sum of the first p terms of an A.P. is equal to the sum of the first q terms, where p ≠ q, then the sum of the first (p + q) terms is:

(A) p + q (B) 0 (C) −(p + q) (D) 1

Q20. Find the number of two digits number that are divisible by 2 or 3

(A) 62 (B) 55 (C) 58 (D) 60

Q21. If the median of the data 24, 27, 28, 31, 2x + 1, 37, 40, 42 (in ascending order) is 32, the value of x is:

(A) 15 (B) 16 (C) 17 (D) 18

Q22. A card is drawn from a well-shuffled deck of 52 cards. The probability that the card is neither a king nor a red card is:

(A) $\frac{3}{13}$ (B) $\frac{6}{13}$ (C) $\frac{7}{26}$ (D) $\frac{9}{26}$

Q23. If each observation of a data set is increased by 5%, then the mean of the data:

(A) Remains unchanged (B) Increases by 5%

(C) Increases by 5 (D) Gets multiplied by 5

Q24. If the total surface area of a solid hemisphere is 462 cm², its volume is:

(A) 718.67 cm³ (B) 739.2 cm³ (C) 654.5 cm³ (D) 812.5 cm³.

Q25. A solid metallic sphere of radius 6 cm is melted and recast into small cones, each of radius 3 cm and height 4 cm. The number of cones formed is:

(A) 6 (B) 12 (C) 24 (D) 36

Q26. If sec θ + tan θ = x, then sin θ =

(A) $\frac{x^2+1}{x^2-1}$ (B) $\frac{x^2-1}{x^2+1}$ (C) $\frac{2x}{x^2+1}$ (D) $\frac{x^2-1}{2x}$

Q27. The angles of elevation of the top of a tower from two points at distances 4 m and 9 m from its base, on the same straight line, are complementary. The height of the tower is:

(A) 5 m (B) 6 m (C) 12 m (D) 36 m

Q28. If a chord of a circle subtends an angle of 60° at the centre, the ratio of the area of the minor segment to the area of the circle is:

(A) $\frac{\pi - 3\sqrt{3}}{6\pi}$ (B) $\frac{2\pi - 3\sqrt{3}}{12\pi}$ (C) $\frac{\pi - \sqrt{3}}{4\pi}$ (D) $\frac{3\pi - 2\sqrt{3}}{6\pi}$

Q29. If two concentric circles have radii 5 cm and 3 cm, the length of a chord of the larger circle which touches the smaller circle is:

(A) 4 cm (B) 8 cm (C) 10 cm (D) 6 cm

Q30. The value of $\tan 1° \cdot \tan 2° \cdot \tan 3° \cdot \ldots \cdot \tan 89°$ is:

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) Not defined

Q31. The ratio of the volume of a cube to the volume of a sphere inscribed in it is:

(A) 6 : π (B) π : 6 (C) 4 : π (D) π : 3

Q32. The sum of all interior angles of a regular octagon is:

(A) 360° (B) 720° (C) 1080° (D) 1440°

Q33. If $x^{\frac{1}{3}} + x^{-\frac{1}{3}} = 2$, then $x^3 - x^{-3} =$

(A) 0 (B) 1 (C) 2 (D) 3

Q34. The curved surface area of a cylinder is 264 m² and its volume is 924 m³. The ratio between its diameter and height is:

(A) 7 : 6 (B) 6 : 7 (C) 3 : 7 (D) 7 : 3

Q35. The value of $\sqrt{20 + \sqrt{20 + \sqrt{20 + \ldots}}}$ is:

(A) 2 (B) 4 (C) 5 (D) 40

Q36. The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each saves ₹2000 per month, one person's monthly income is:

(A) ₹18,000 (B) ₹16,000 (C) ₹9,000 (D) ₹20,000

Q37. The ratio of length to perimeter of a rectangle is 5 : 16. The ratio of its length to width is:

(A) 5 : 11 (B) 5 : 8 (C) 5 : 4 (D) 5 : 3

Q38. The ratio of the sides of two regular polygons is 1 : 2 and the ratio of their interior

angles is 2 : 3. The numbers of sides of the polygons are:

(A) 4, 8 (B) 5, 10 (C) 6, 12 (D) 8, 16

Q39. How many two-digit natural numbers are divisible by 3?

(A) 60 (B) 30 (C) 50 (D) 20

Q40. Which statement is true for the discriminant of the quadratic equation $ax^2 + bx + c = 0$?

(A) Two distinct real roots if $b^2 - 4ac > 0$ (B) Two equal real roots if $b^2 - 4ac = 0$

(C) No real roots if $b^2 - 4ac < 0$ (D) All of the above

Q41. If two similar triangles have corresponding sides in the ratio 3 : 5, their perimeter are in the ratio:

(A) 3 : 5 (B) 6 : 10 (C) 9 : 25 (D) 25 : 9

Q42. If the mean of the first n natural numbers is $\frac{5n}{9}$, n =

(A) 5 (B) 4 (C) 9 (D) 10

Q43. A solid consists of a right circular cylinder surmounted by a right circular cone. The height of the cone is h. If the total volume of the solid is 3 times the volume of the cone, the height of the cylinder is:

(A) 2h (B) $\frac{3h}{2}$ (C) $\frac{h}{2}$ (D) $\frac{2h}{3}$

Q44. If $\sin(9° - \theta) = \sin\left(\frac{k}{3} - \theta\right)$, k =

(A) $43\frac{1}{2}°$ (B) 54° (C) 27° (D) $13\frac{1}{2}°$

Q45. In an equilateral triangle ABC, if AD ⊥ BC, then:

(A) $5AB^2 = 4AD^2$ (B) $3AB^2 = 4AD^2$ (C) $4AB^2 = 3AD^2$ (D) $2AB^2 = 3AD^2$

Q46. The number of quadratic equations having real roots whose roots do not change by squaring them is:

(A) 4 (B) 3 (C) 2 (D) 1

Q47. If 217x + 131y = 913 and 131x + 217y = 827, then x + y =

(A) 5 (B) 6 (C) 7 (D) 8

Q48. If $f(x) = ax^2 + bx + c$ has no real zeroes and a + b + c < 0, then:

(A) c = 0 (B) c > 0
(C) c < 0 (D) c can be positive or negative

Q49. If n is a natural number, then $9^{2n} - 4^{2n}$ is always divisible by:
(A) 5 (B) 13 (C) Both 5 and 13 (D) None of these

Q50. The LCM of the smallest odd prime number and the greatest two-digit number is:
(A) 1 (B) 99 (C) 297 (D) 300

Q51. $n^2 - 1$ is divisible by 8 if n is:
(A) An integer (B) A natural number
(C) An odd integer (D) An even integer

Q52. The sum of n terms of the series $\sqrt{3} + \sqrt{12} + \sqrt{27} + \sqrt{48} +$ ... is:
(A) $\frac{2n(n+1)}{\sqrt{3}}$ (B) $\frac{\sqrt{3}n(n-1)}{2}$
(C) $\frac{\sqrt{3}n(n+1)}{2}$ (D) $\frac{2n(n-1)}{\sqrt{3}}$

Q53. The distance of point A from the x-axis is 3 units. Which of the following cannot be the coordinates of A?
(A) (1, 3) (B) (−3, −3) (C) (−3, 3) (D) (−3, 1)

Q54. If $7\cos^2\theta + 3\sin^2\theta = 4$, θ =
(A) 30° (B) 45° (C) 60° (D) 90°

Q55. If an arc subtends an angle of 90° at the centre of a circle, the ratio of its length to the circumference of the circle is:
(A) 1 : 4 (B) 1 : 2 (C) 1 : 3 (D) 1 : 6

Q56. If α, β are roots of $ax^2 + bx + c = 0$, then $\frac{1}{(a\alpha+b)^2} + \frac{1}{(a\beta+b)^2}$ is:
(A) $\frac{b^2-2ac}{ac}$ (B) $\frac{b^2-2ac}{a^2c^2}$ (C) $\frac{2ac-b^2}{ac}$ (D) $\frac{b^2}{a^2c}$

Q57. The diameter of a wheel is 63 cm. The distance travelled by the wheel in 100 revolutions is:
(A) 99 m (B) 198 m (C) 63 m (D) 136 m

Q58. **If four times the sum of the areas of the two circular faces of a cylinder of height 8 cm is equal to twice its curved surface area, then the diameter of the cylinder is:**

(A) 4 cm (B) 8 cm (C) 2 cm (D) 6 cm

Q59. **If the distance between the points (3, −5) and (x, −5) is 15 units, the values of x are:**

(A) +12, −18 (B) −12, 18 (C) 18, 5 (D) −9, −12

Q60. **The perimeter of the triangle with vertices (0, 4), (0, 0) and (3, 0) is:**

(A) $7 + \sqrt{5}$ (B) 5 (C) 10 (D) 12